COMSATS Institute of Information and technology Vehari campus



Department of Mathematics

Assingment#01

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Subject: mechanics ᵢᵢ

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Q#1

A particle starts from O at t=0. Find it, velocity and acceleration at any time “t” if is position at that time is given by

1. =(t3+2t)+(5t2 -7)
2. =at2+4at
3. = acost+ bsint
4. = a(t-cost) +a(1+sint)

Solution:

Considering a particle which starts moving from point O at t=0. Let at any instant the particle is at point p whose position vector w.r.t origin is given by

1: = (t3+2t) +(5t2-7)

Diff. w.r.t ‘t’

d/dt=(3t2 +2(1))+(10t-0)

=(3t2 +2)+10t is the velocity of the particle at any instant ‘t’

Again diff. w.r.t ‘t’

d/dt =(6t+0)+10(1)

= 6ti + 10j is the acceleration of the particle at any instant ‘t’.

2: =at2+4at

Diff. w.r.t ‘t’

d/dt=2at+4a(1)

=2at+4a is the velocity of the particle at any instant ‘t’

Again diff. w.r.t ‘t’

d/dt =2a(1)+0

= 2a is the acceleration of the particle at any instant ‘t’.

3:= acost+ bsint

Diff. w.r.t ‘t’

d/dt= a(-sint)+ bcost

= -asint+ bcost is the velocity of the particle at any instant ‘t’

Again diff. w.r.t ‘t’

d/dt = -acost+ b­(-sint)

= -(acost+ bsint) is the acceleration of the particle at any instant ‘t’.

4:= a(t-cost) +a(1+sint)

Diff. w.r.t ‘t’

d/dt= a((1)-(-sint))+ a(0+cost)

= a(1+sint)+ acost is the velocity of the particle at any instant ‘t’

Again diff. w.r.t ‘t’

d/dt = a(0+cost)+ a(-sint)

= acost- asintis the acceleration of the particle at any instant ‘t’.

Q#02

The position of a particle moving along an ellipse is given by =acosti + bsintj

If a>b, find the position of the particle where velocity has a maximum aor minimum magnitude.

Solution:

Considering a particle which is moving along an ellipse let at any time ‘t’ x’ the particle is at point (r,) whose position vector w.r.t ‘O’ is .

y

p(r,)

Then= acost +bsint

a>b

b

N

Diff. w.r.t. ‘t’

Y’

o

X’

x

a

d/dt=a(-sint)+bcost

=-asint+bcost

||=

v=

v2=a2sin2t+b2cos2t

since for the velocity ‘v’ to be maximum

1. =0
2. 0

And for the velocity ‘v’ to be miximum

1. =0
2. 0

Agin

V2=a2sin2t+b2cos2t

Diff. w.r.t. ‘t’

2(v)’=a2(2sintcost)+b2(2cost(-sint))

2v=a2sin2t-b2sin2t

2v=(a2-b2)sin2t

2v(0)=(a2-b2)sin2t

(a2-b2)sin2t=0

Sin2t=0 a2-b2≠0 as a>b

2t=0,π,2π,3π

/2 t=0,, π,

Again

2v=(a2-b2)sin2t

Diff. w.r.t ‘t’

2(v+.)=(a2-b2)cos2t.2 , v=

v+()2=(a2-b2)cos2t , put t=0

1. put t=0, =0, v=b , then v=b

b+0=(a2-b2)cos2(0)

=

0 as a>b

Clearly for t=0; (1) =0 (2) 0 (1)

Again

v+()2=(a2-b2)cos2t v=

put t=, =0 ,v=a put t=

a+0=(a2-b2)cos2 then v=a

=

=0 ,v=a

Clearly for t=0; (1) =0 (2) 0 (2)

Again

v+()2=(a2-b2)cos2t , v=

put t=, =0 ,v=b , put t=

b+0=(a2-b2)cos2 , then v=b

=

0, as v=b

Cleary for t=, (1)=0 (2) 0 (3)

Again

v+()2=(a2-b2)cos2t , v=

put t=, =0 ,v=a , put t=

b+0=(a2-b2)cos2 , then v=a

=

0, as v=b

Cleary for t=, (1)=0 (2) 0 (4)

Clearly the velocity will be minimum for

t=o,π,…………(from eq 1 and 3)

now

=acost +bsint

put t=o,π

we get =a,-a

=±a are the positions of the particle where it attain minimum velocity ,clearly will be maximum for

t=,,……….(form eq 2 and 4)

now

=acost+bsint

put t=

we get=b,-b

r=±bj are the positions of the particle where it attains its maximum velocity.

Q#03

A particle is moving with uniform speed ‘v’ along the curve .

x2y=a(x2+)

show that its acceleration has the maximum value 10v2/9a

solution :

considering a particle which is moving along the curve x2y=a(x2+)

with constant speed ‘v’

then =0

so, the tangential component of =0 and the normal component of acceleration =v2/s

now

x2y=a(x2+)

deviding by x2

y=a(1+x-2)

diff . w.r.t ‘x’

=a(0+(-2)x-3)

y1=x-3

again diff .w.r.t ‘x’

=-2(-3)x-4

Y2=x-4

Now

=(1+Y21)3/2=

Now the normal component of acceleration=V2/

A=

=

=

Since ‘A’ depends on B=x5

‘A’ will be maximum if ‘B’ is maximum

Now for B to be maximum

1. =0 (2)

Now

B= x5

Diff . w.r.t ‘x’

=

=

Put =0

=0

,x=0 ||

4x6=4a6

x6=a6, x=a

again

Diff . w.r.t ‘x’

=

When x=0 , =

When x=a

=

i.e.

clearly for x=a

1. =0 (2)

Clearly for x=a ,B=will maximum. hence A= will be maximum.

Now

A= put x=a

A=

A= ==as required.

Q#04

Find the tangential and normal components of acceleration of a point deseribing the ellipre

+=1

With uniform speed ‘v’ when the particle is at(o,b).

Solution:

Considering a point which is moving along the ellipse +=1 with uniform (constant) speed v then

multiply a2b2

b2x2+a2y2=a2b2

diff. w.r.t ‘x’

b2(2x)+a2(2y)=0

b2x+a2y=0

a2y= -b2x

= (x,y)=(0,b)

Put x=0,y=b

y1 = =0

since y1=

diff. w.r.t ‘x’

= ; (x,y)=(0,b)

Put x=0 , y=b

Y2=

Y2=

Now

Normal component of acceleration===-

Q#05.

Find the radial and transverse components of the velocity of a particle moving along the curve ax2+by2=1 at any time ‘t’ if the polar angle

Solution:

Considering a particle which is moving along the curve ax2+by2=1 (1)

We know that relation connecting Cartesian and polar coordinates are

X=rcos ,y=rsin

So from (1) we have

A(rcos)2+b(rsin)2=1

ar2cos2 +br2sin2 =1

r2(acos2 +bsin2)=1

r2=

r=

r=(acos2 +bsin2)-1/2

the radial component of velocity =dr/dt

the transverse component of velocity=r

now

r=(acos2 +bsin2)-1/2

diff . w.r.t ‘t’

=

, ,

Radial component of velocity=

Transverse component of velocity =

=

=

Q#0**6**

Find the radial and transverse components of acceleration of a particle moving along a circle x2+y2=a2 with constant angular velocity c.

Solution:

Considering a particle which is moving along a circle

1

X2+y2=a2 (1)

We know that the sedation connecting Cartesian and polar coordinates are

X=rcos ;=rsin

So form eq 1 we have

The radial component of acceleration

The transverse component of

Acceleration

=

Angular velocity =c

i.e

=2(0)(c)+a(0)

=0+0=0